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VV256 RC2 Part2

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Chain Rule:

具体来说，如果有两个函数 f 和 g，且 $y = f(g(x))$，那么链式法则表明：

$$\frac{dy}{dx} = \frac{dy}{dg} \cdot \frac{dg}{dx}$$

这里：

- $\frac{dy}{dg}$ 是 f 关于 g 的导数。
- $\frac{dg}{dx}$ 是 g 关于 x 的导数。

Chain Rule:

Q1: solve $\frac{dy}{y} = 2x \frac{1}{\cos(y^2)} dx$ $\quad \frac{dy}{dx} = \frac{2xy}{\cos(y^2)}$

Let $u = y^2$ $\quad \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$\downarrow$

$y = \sqrt{u}$ $\quad \frac{dy}{du} = \frac{1}{2\sqrt{u}}$

$\frac{2xy}{\cos(y^2)} = \frac{1}{2\sqrt{u}} \frac{du}{dx}$ $\quad \frac{2x\sqrt{u}}{\cos u} = \frac{1}{2\sqrt{u}} \frac{du}{dx}$

$\frac{du}{dx} = \frac{4xu}{\cos u}$ $\quad 4x\,dx = \frac{\cos u}{u} du$

$\int \frac{\cos u}{u}$ $\quad \frac{\cos u}{u} + \frac{\cos(-u)}{-u} = 0$

surprise!!! $\quad 4 \cdot \frac{1}{2} x^2 = C$ $\quad 2x^2 = C$

Method of changing variable

Q2: $y^2(1-y')=(2-y')^2$ 构造齐次项

$y^2 \sim (2-y')^2$ 构造 $y \sim 2-y'$ 的关系

Let

$2-y'=yt$ ① $y^2(1-y')=(2-y')^2$

$y^2(1-y')=y^2t^2$ $1-y'=t^2$ $y'=1-t^2=\frac{dy}{dx}$

$ty=2-1+t^2=1+t^2$

$y=t+\frac{1}{t}$ $\frac{dy}{dt}=1-\frac{1}{t^2}$

$y'=\frac{dy}{dx}$ $dx=\frac{1}{y'}dy=\frac{1}{y'}\frac{dy}{dt}dt=\frac{1}{1-t^2}\left(\frac{t^2-1}{t^2}\right)dt$

$=-\frac{1}{t^2}dt$

$\begin{cases} x=\frac{1}{t}+C \\ y=\frac{1}{t}+t \end{cases}$ OR $y=x-C+\frac{1}{x-C}$

And if $y'=0$ $y=2$ $y^2=4$ $y=\pm 2$

Method of undertermined coefficient 不常规的（Or Guess）

Q3: Also ~ Guess

Last Fall: we guess the solution is $e^{\lambda t}$

but most students have no idea

I guess t^{λ} with some points

easy question:

$$t^2 y'' - 3t y' + 4y = 0$$

$t^2 \sim ''$

$t \sim '$

$1 \sim$

guess $y = t^{\lambda}$ Then $\Rightarrow$ easy

“配凑”

Q4:

$\int x^2 \sin x$.

① 配凑 we know $(fg)' = f'g + fg'$

In order to get $x^2 \sin x$

we need $\boxed{x^2\cos x}$

but it ↙ produce $2x \cdot \sin x$ ~

we need to produce sth to overcume $2x\sin x$ ~

...

$\int x^2 \cos x$ $\qquad (x^2 \sin x)' = 2x\sin x + x\cos x$

Then

$(x^2 \sin x)' - 2x\sin x = x\cos x$ --- step by step

we need to produce sth to overcume $2x\sin x$ ~

...

$\int x^2 \cos x$ $\qquad (x^2 \sin x)' = 2x\sin x + x\cos x$

Then

$(x^2 \sin x)' - 2x\sin x = x\cos x$ --- step by step

Final

$\int x^2 \cos x = \Big[\underbrace{x^2 \sin x + A x\cos x + B\sin x}_{= t(x)}\Big]$

求导 $t'(x) = 2x\sin x + x^2\cos x + A[\cos x - x\sin x] + B\cos x$ ---

Integration by parts

② 分部

$$\int x^2\cos x\,dx = \int x^2\,d(\sin x) = x^2\sin x - \int \sin x\,d(x^2)$$

$$= x^2\sin x - \int 2x\sin x\,dx$$

$$\int x\sin x\,dx = -\int x\,d(\cos x) = -\left[x\cos x - \int \cos x\,dx\right]$$

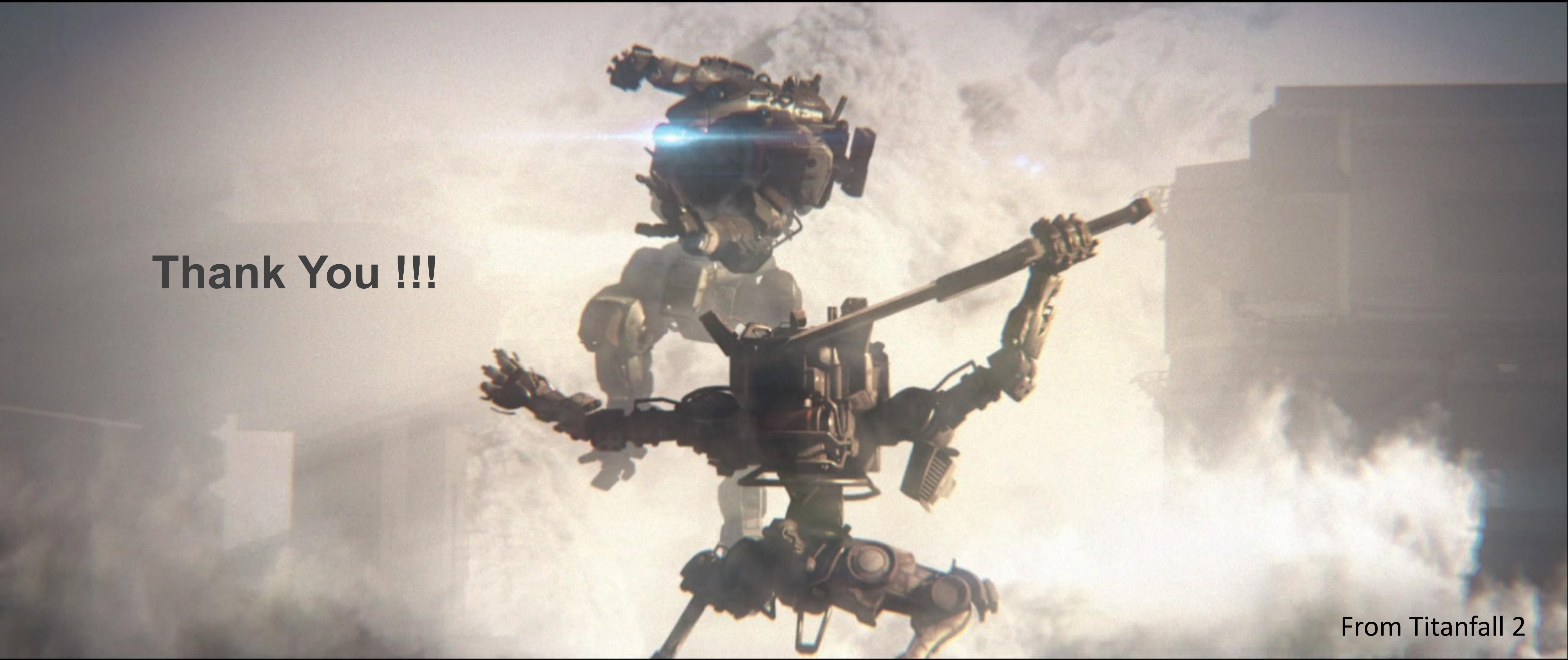
...

easy? But in mid/final exam

多项式 like $\int x^n\sin^k x + x^m\cos^t x \cdots$

appear last year

Thank You !!!



From Titanfall 2